USN

Fifth Semester B.E. Degree Examination, May/June 2010

Modern Control Theory

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define the following terms : i) State ii) State variables iii) State vector iv) State space. **(06 Marks)**

b. For the electrical network shown in figure Q1 (b), choose V_Q, V_R and i_L as state variables and derive the state space model. Voltage across the C_2 is output. **(08 Marks)**

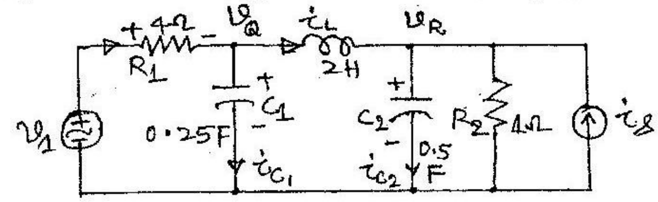

Fig. Q1 (b)

c. Obtain the state model of the following system, responded by the following differential equation, $\dddot{y} + 6\ddot{y} + 5\dot{y} + y = u(t)$. Draw the block diagram. **(06 Marks)**

2 a. Obtain the Jordan form of the canonical state model for the giveri transfer function and draw the block diagram.

$$\frac{y(s)}{u(s)} = \frac{2s^2 + 6s + 7}{(s+1)^2(s+2)}$$ **(08 Marks)**

b. Derive the transfer function from the state space model. **(04 Marks)**

c. Consider $\dot{x} = Ax + Bu$ and $y = Cx$ with $A = \begin{bmatrix} 0 & 2 & 0 \\ 4 & 0 & 1 \\ -48 & -34 & -9 \end{bmatrix}$ **(08 Marks)**

Determine : i) Characteristic equation ii) Eigen values iii) Eigen vectors iv) Modal matrix.

3 a. Write the properties of state transition matrix. **(05 Marks)**

b. Find the state transition matrix by using Cayley Hamilton method for $A = \begin{bmatrix} 0 & 2 \\ -2 & -4 \end{bmatrix}$. **(05 Marks)**

c. Determine the STM (State Transition Matrix) by Laplace inverse method and find the solution of the system described by the vector differential equation:

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} x$$

Assume the system to be relaxed initially i.e. $x_1(0) = x_2(0) = 0$. **(10 Marks)**

4 a. Define controllability and observability. Discuss duality of controllability and observability. **(06 Marks)**

4 b. Derive controllability test by Gilbert's method. **(08 Marks)**

c. Examine the observability of the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u,\ y = [4 \quad 5 \quad 1]\dot{x}$$ **(06 Marks)**

PART – B

5 a. With reference to a non linear system, explain the following, with the help of a figure and input output characteristics: i) Jump response ii) Backlash. **(08 Marks)**

b. Define singular points. List the singular points based on the locations of eigen values of the system, along with the phase portraits. **(06 Marks)**

c. Plot eigen values and phase portraits of the system expressed by the following equations:

i) $\ddot{y} + a\dot{y} + 20y = 0$ ii) $\ddot{y} - 8\dot{y} + 17y = 34$ **(06 Marks)**

6 a. Define a controller. Explain the effects of integral and derivative control actions, on system performance, for a second order system, subjected to unit step input. **(08 Marks)**

b. A system is described by the following state space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -6 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u$$

Design a state feedback controller such that the desired closed poles are at $(-1+j)$, $(-1-j)$, (-5). **(06 Marks)**

c. Consider the system, $\dot{x} = Ax + Bu$, $y = Cx$ with $A = \begin{bmatrix} 0 & 10 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [0 \quad 1]$. Design a full order state observer to implement the state feedback $u = -k\tilde{x}$ with eigen values of the observer to be at -10, 10. **(06 Marks)**

7 a. Define : i) Stability in the sense of Liapunov ii) Asymptotic stability iii) Asymptotic stability in the large. **(06 Marks)**

b. Examine the stability of the system described by the following equation by Krasovaskii's theorem:

$\dot{x}_1 = -x_1$ and $\dot{x}_2 = x_1 - x_2 - x_2^3$ **(08 Marks)**

c. Discuss the concept of Liapunov's first and second method of stability theorem. **(06 Marks)**

8 a. Consider the system with differential equation, $\ddot{e} + K\dot{e} + K_1\dot{e}^3 + e = 0$. Examine the stability by Liapunov's method, given K>0 and K_1>0. **(08 Marks)**

b. A system is described by the following:

$\dot{x} = Ax$, where $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$

Assume matrix Q to be identity matrix; solve for matrix D in the equation, $A^T P + PA = -Q$. Use Liapunov theorem and determine the stability of the origin of the system. Write the Liapunov function $\overline{V}(x)$. **(12 Marks)**

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